Module - 4

Communication through Band limited channels

For base band data transmission, transmitted pulses tend to spread during transmission. The pulse spreading or pulse dispersion causes overlapping of pulses into adjacent time slots. This may result in an error at the receiver.

"The phenomenon of pulse spreading & resultant difficulty in discriminating the symbols at the receiver is termed as Inter symbol Interference [ISI]."

** Digital PAM transmission through Band limited channels :-

The binary data $\{b_k\}$ is applied to the pulse generator which generates the signal $x(t)$.

i.e., $$x(t) = \sum_{k=-\infty}^{\infty} A_k \, g(t - kT_b)$$ where,

$T_b \rightarrow$ duration of each binary bit.

$g(t) \rightarrow$ is the shaping pulse normalized to 1V.

i.e., $g(0) = 1V$.

$$A_k = \begin{cases} +a & \text{if } b_k = 1 \\ -a & \text{if } b_k = 0 \end{cases}$$

The PAM signal $x(t)$ is then passed through the transmitting filter of transfer function $H_T(f)$. The signal is then passed through the channel having the transfer function $H_c(f)$. The channel may be co-axial cable or optical fibre.

The channel o/p is passed through a receiving filter of transfer function $H_R(f)$. The o/p of the rxg filter is $y(t)$. This $y(t)$ is noisy replica of the transmitted signal $x(t)$.

The o/p $y(t)$ is sampled with the sampling instants $t = i T_b$. The sampled signal $y(iT_b)$ is given to the decision device. The decision device compares the i/p signal with threshold 'λ'

if $y(iT_b) > \lambda$ symbol is '1'

if $y(iT_b) < \lambda$ symbol is '0'

The receiving filter output is,

$$y(t) = \mu \sum_{K=-\infty}^{\infty} A_K \, P(t - KT_b)$$

$\mu \rightarrow$ Scaling factor

$P(t) \rightarrow$ normalized pulse i.e., $P(0) = 1$.

@ $t = iT_b$

$$y(iT_b) = \mu \sum_{K=-\infty}^{\infty} A_K \, P[iT_b - KT_b]$$

$$= \mu \sum_{K=-\infty}^{\infty} A_K \, P[(i-K)T_b]$$

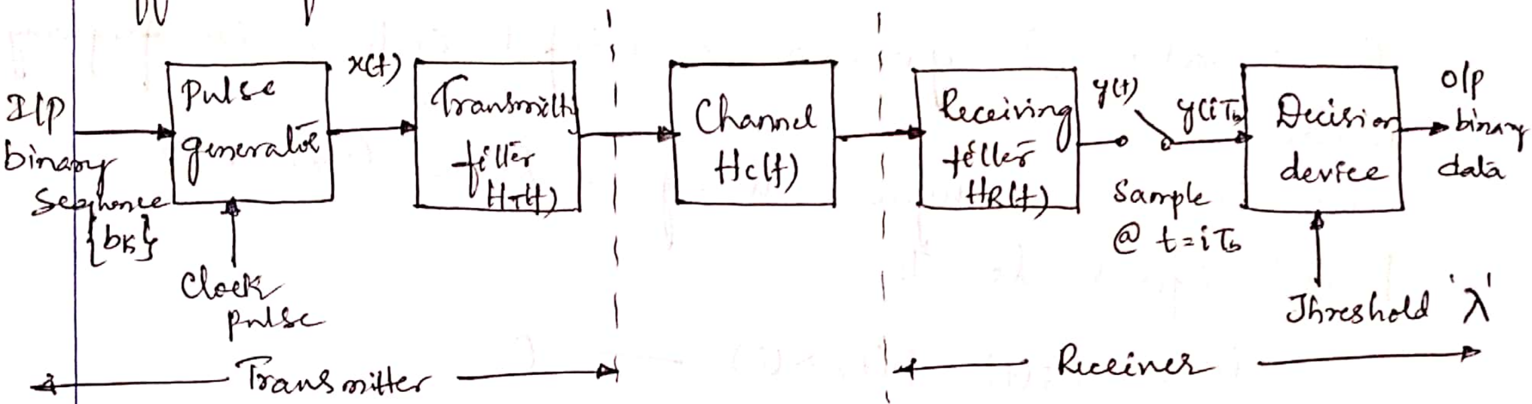
$$y(iT_b) = \mu A_i P(0) + \mu \sum_{\substack{K=-\infty \\ K \neq i}}^{\infty} A_K \, P[(i-K)T_b]$$

The first term represents the i^{th} transmitted symbol. The 2nd term represents the residual effect of all other bits transmitted before & after the sampling instants iT_b. This residual effect is called Inter Symbol Interference [ISI].

In the absence of ISI

$$y(iT_b) = \mu A_i$$

The presence of ISI introduces error at the receiver output. Therefore while designing the transmitting & receiving filters it is necessary to minimize the effects of ISI.



The effect of ISI can be reduced by following methods

1) Nyquist pulse shaping criterion.
2) Raised cosine spectrum.
3) Correlative coding.

Nyquist Criterion for distortionless base band binary transmission [Zero ISI] :-

Consider the equation,

$$y(iT_b) = \mu A_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k \, p[(i-k)T_b] \quad \text{--- (1)}$$

In the above equation, the second term must be zero to eliminate the effect of ISI. This is possible, if the received pulse p(t) is controlled such that,

$$p[(i-k)T_b] = \begin{cases} 1 & \text{for } i=k \\ 0 & \text{for } i \neq k \end{cases} \quad \text{--- (2)}$$

If $p(t)$ satisfies the above eqn (condition), then we get an o/p which is free from ISI.

This condition is known as Nyquist criterion for zero ISI.

$\therefore\ y(iT_b) = \mu A_i$ for $i \neq k$. which implies zero ISI.

The equation ② gives more useful criteria in frequency domain.

If $p(t)$ is sampled using a Dirac Comb with a period equal to T_b.

i.e., $P_\delta(t) = p(t)\,\delta(t)$ ——— ③

$$= p(t) \sum_{m=-\infty}^{\infty} \delta(t - mT_b)$$

$$= \sum_{m=-\infty}^{\infty} p(mT_b)\,\delta(t - mT_b)$$

Taking FT on both sides of eqn ③,

$$P_s(f) = P(f) * \delta(f)$$

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - nf_b) \text{ ——— ④}$$

From the definition of F.T

$$P_\delta(f) = \int_{-\infty}^{\infty} P_\delta(t)\, e^{-j2\pi ft}\, dt.$$

$$P_s(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} p(mT_b)\, \delta(t - mT_b)\, e^{-j2\pi ft}\, dt.$$

Let the integer $m = i - k$ then $i = k$ corresponds to $m = 0$ lly $i \neq k$ corresponds to $m \neq 0$.

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i.e., eqn ② becomes

$$P[(i-k)T_b] = P(mT_b) = \begin{cases} 1, & m=0 \\ 0, & m\neq 0 \end{cases}$$

For $i=k$ i.e., $m=0$

$$P_s(f) = \int_{-\infty}^{\infty} P(0)\,\delta(t)\,e^{-j2\pi ft}\,dt$$

For $i\neq k$ i.e., $m\neq 0$

$$P_s(f) = 0$$

But from the defn of FT of a delta fun.

i.e.,
$$\int_{-\infty}^{\infty} \delta(t)\,e^{-j2\pi ft}\,dt = 1$$

$$\therefore \boxed{P_s(f) = P(0)} \quad \text{for } i=k.$$

$\Rightarrow$ $P_s(f) = 1$ for normalized value of $P(0)=1$

Equation ④ becomes,

$$1 = f_b \sum_{n=-\infty}^{\infty} P(f-nf_b)$$

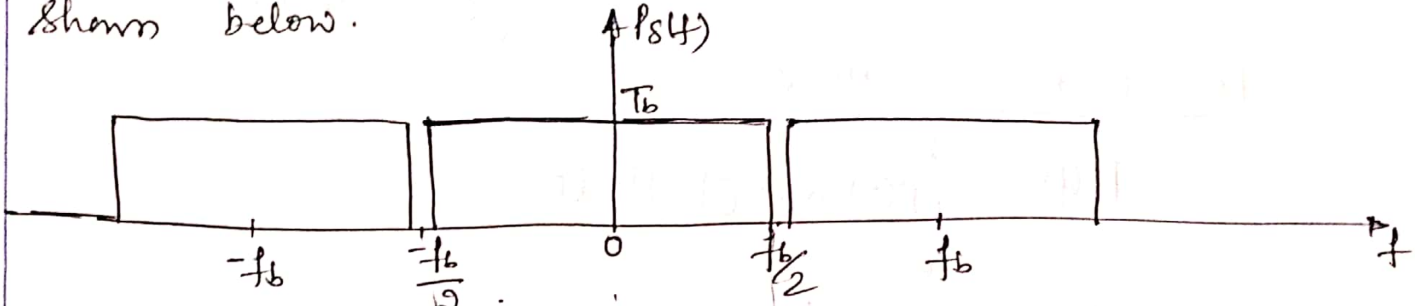
or
$$\boxed{\sum_{n=-\infty}^{\infty} P(f-nf_b) = \frac{1}{f_b} = T_b} \quad \text{——⑤}$$

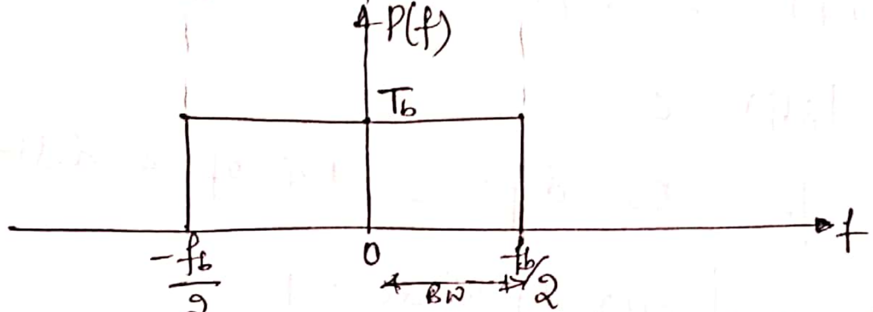
This is the frequency domain condition for zero ISI & this condn is called Nyquist criterion for distortionless base band transmission.

Ideal Solution or Nyquist soln for zero ISI :-

The effect of ISI can be minimized by controlling $p(t)$ in time domain & $P(f)$ in frequency domain.

Equation (5) represents the spectrum which repeats with period f_b & it has amplitude of T_b. This spectrum is shown below.





P(f) shows the spectrum of the expected signal p(t). It can be represented by rect function as

$$P(f) = \frac{1}{f_b}\,\text{rect}\left(\frac{f}{f_b}\right)$$

$f_b = 2B_0$

Inverse FT of above function is

$$\frac{1}{f_b}\,\text{rect}\left(\frac{f}{f_b}\right) \longleftrightarrow \text{Sinc}(f_b t).$$

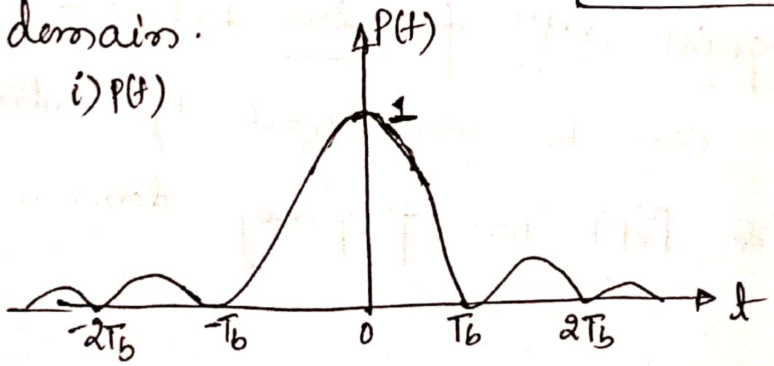
i.e., $p(t) = \text{Sinc}(f_b t)$

The B.W of the pulse can be represented by $B_0 = \frac{f_b}{2}$.

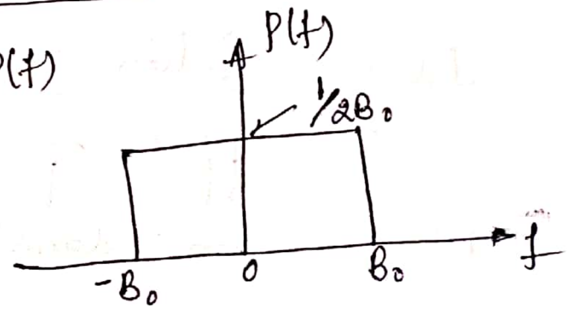
$$\therefore\ p(t) = \text{Sinc}(2B_0 t).$$

The functions which gives zero ISI are $\boxed{p(t) = \text{Sinc}(2B_0 t)}$ in time domain & $\boxed{P(f) = \frac{1}{f_b}\,\text{rect}\left(\frac{f}{f_b}\right)}$ in frequency domain.

i) p(t)



ii) P(f)



Nyquist B.W is given by,

$$\boxed{B_0 = \frac{f_b}{2} = \frac{1}{2T_b}}$$

$f_b = 2B_0$

"Nyquist B.W is defined as the minimum transmission B.W for zero ISI."

$$P(f) = \frac{1}{2B_0} \text{ rect}\left(\frac{f}{2B_0}\right).$$

on simplification (using rect property).

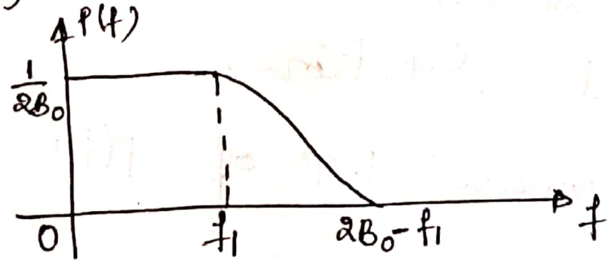
$$P(f) = \begin{cases} \frac{1}{2B_0} & \text{for } -B_0 \leq f \leq B_0 \\ 0 & \text{else where.} \end{cases}$$

The spectrum of P(f) is flat from $-B_0$ to $+B_0$ & zero elsewhere. since There is an abrupt transitions in the frequency at $\pm B_0$. Such frequency spectrum is not physically realizable. Hence the ideal sinc pulse cannot be physically generated.

* Raised Cosine pulse shaping :-

In the raised cosine spectrum, the frequency response P(f) decreases towards zero gradually (i.e., there is no abrupt transition).

In otherwords, P(f) should have a flat portion & a roll-off portion that has the form of a raised cosine spectrum as shown below,



Mathematically, a raised cosine spectrum is defined as.

$$P(f) = \begin{cases} \frac{1}{2B_0} & \text{for } 0 \le f \le f_1 \\ \frac{1}{4B_0}\left[1+\cos\left(\frac{\pi(|f|-f_1)}{2B_0-2f_1}\right)\right], & \text{for } f_1 \le |f| < 2B_0 - f_1 \\ 0, & |f| \geq 2B_0 - f_1 \end{cases}$$

In the above equation, the middle term represents the gradual roll off from f_1 to $2B_0 - f_1$. The freq. f_1 & the Nyquist B.W B_0 are related by,

$$\alpha = 1 - \frac{f_1}{B_0}$$

The parameter α is called roll-off factor.

The frequency spectrum $P(f)$ is plotted for different values of α i.e, $\alpha = 0$, 0.5 & 1 as shown below,

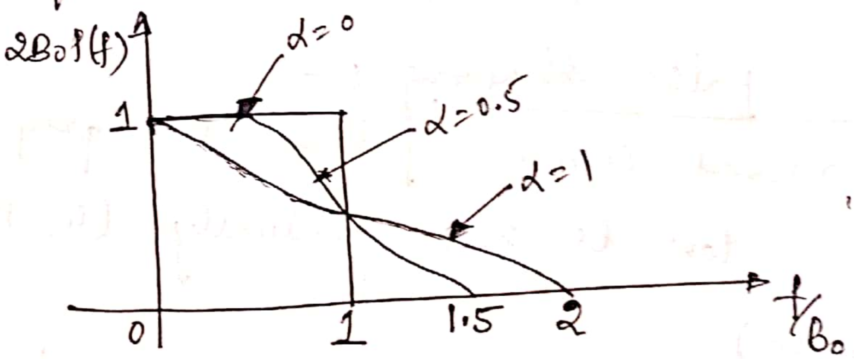


Fig: Amplitude response for different roll-off factors.

For $\alpha=1$, we get full cosine roll-off characteristics with the transmission b.w at $2B_0$ which is twice that of ideal solution.

The inverse F.T of $P(f)$ gives.

$$P(t) = \text{Sinc}(2B_0t)\,\frac{\cos(2\pi\alpha B_0 t)}{1-16\alpha^2B_0^2t^2}$$

When $\alpha = 1$, we get-

$$P(t) = \frac{\text{Sinc}(4B_0t)}{1-16B_0^2t^2}$$

$$P(f) = \begin{cases} \frac{1}{4B_0}\left[1+\cos\left(\frac{\pi f}{2B_0}\right)\right], & 0 < |f| < 2B_0 \\ 0, & |f| \geq 2B_0 \end{cases}$$

Transmission B.W requirement:-

The frequency response P(f) is non zero in the interval $[0, (2B_0 - f_1)]$. The transmission b.w is

$$B = 2B_0 - f_1$$

where $B_0 = \frac{1}{2T_b}$ is the Nyquist B.W & $\alpha = 1 - \frac{f_1}{B_0}$

$$f_1 = B_0(1-\alpha)$$

$$B = 2B_0 - B_0(1-\alpha)$$

$$\boxed{B = B_0(1+\alpha)}$$

For $\alpha = 0$, $B = B_0$, this is same as ideal solution. when $\alpha = 1$,

As α increases, the B.W increases, when $\alpha = 1$, $B = 2B_0$ i.e., bandwidth required by the raised cosine spectrum is double of the Nyquist B.W.

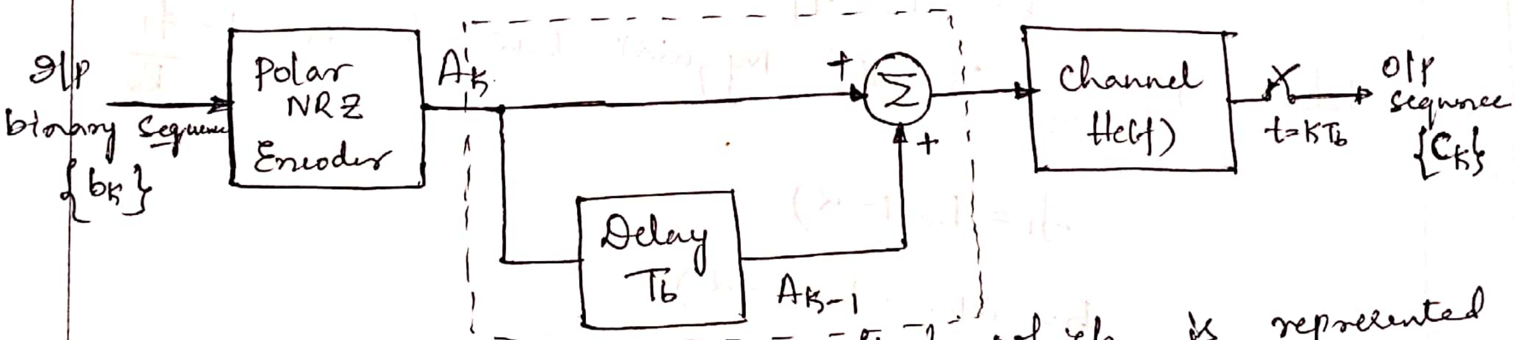
** Correlative Coding :-

In correlative coding two successive bits are used for evaluation. It is also called duobinary encoding.

Consider two successive pulses $p(t)$ and $p(t-T_b)$. If both $p(t)$ & $p(t-T_b)$ are positive, the sample value will be 2. If both $p(t)$ & $p(t-T_b)$ are negative, the sample value will be -2. If $p(t)$ & $p(t-T_b)$ are of opposite polarities, the sample value will be 0. This gives us the logic to make correct decisions.

The correlative coding is implemented by duobinary signaling & modified duobinary encoding.

* Duo binary Encoding :-



Consider the input sequence $\{b_k\}$ which is represented by polar NRZ sequence A_k.

i.e., $$\begin{cases} A_k = +1V, & \text{If } b_k = 1 \\ A_k = -1V, & \text{If } b_k = 0 \end{cases}$$

When this sequence is applied to a duobinary encoder, the output will contain three levels -2, 0 and $+2$. The o/p of the duo binary encoder can be expressed as,

$$C_k = A_k + A_{k-1}$$

Here the uncorrelated sequence $\{A_k\}$ of two level pulses is converted into correlated sequence of three level pulses.

Reconstruction: Let $\hat{A}_k$ represents the estimate of A_k. Then we can obtain $\hat{A}_k$ as

$$\hat{A}_k = C_k - \hat{A}_{k-1}$$

This shows that if C_k is received with error, then $\hat{A}_k$ will have error. This error will propagate in the output sequence.

Drawbacks :- Error propagation takes place in the decoder.

Example :-

Sequences K	0	1	2	3	4	5	6	7	8
1) Binary seq b_k	0	0	1	1	0	1	0	0	1
2) polar representation A_k	-1	-1	+1	+1	-1	+1	-1	-1	+1
3) Duo binary encoder o/p $C_k = A_k + A_{k-1}$	-	-2	0	+2	0	0	0	-2	0
4) Estimate of $\hat{A}_k = C_k - \hat{A}_{k-1}$	-1	-1	+1	+1	-1	+1	-1	-1	+1
5) Estimated binary sequence.	0	0	1	1	0	1	0	0	1

** Duo binary Encoder with Precoder :-

Encoder :- Precoder is used in duobinary encoder to avoid error propagation. Fig shows the block diagram of duobinary encoder with precoder. Precoder is nothing but a differential encoder.

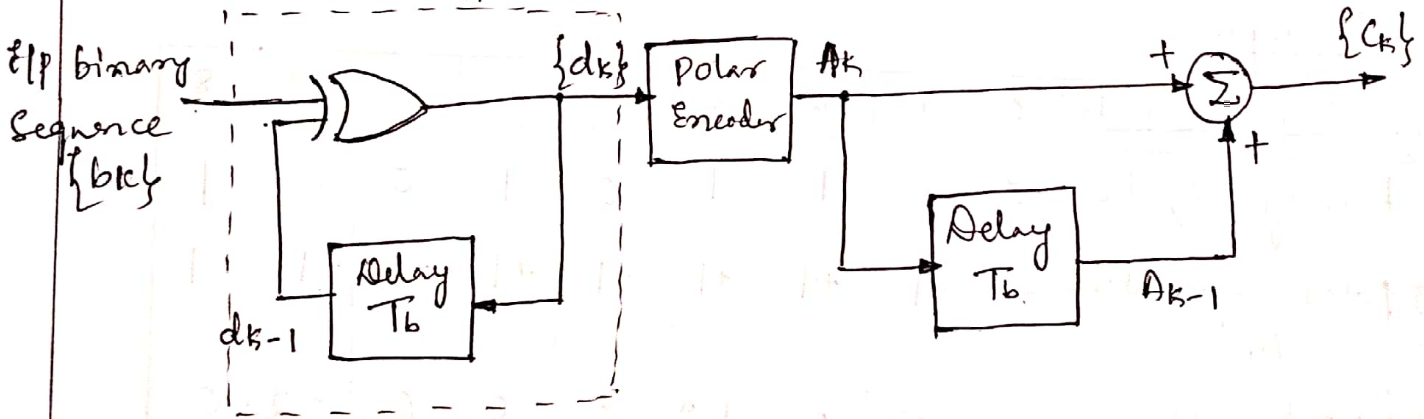
The O/p of the precoder is

$$d_k = b_k \oplus d_{k-1}$$

This is normal exclusive-OR operation.

The precoded sequence $\{d_k\}$ is applied to polar NRZ level encoder. The sequence $\{A_k\}$ is then applied to duobinary encoder. The output C_k is

$$C_k = A_k + A_{k-1}$$

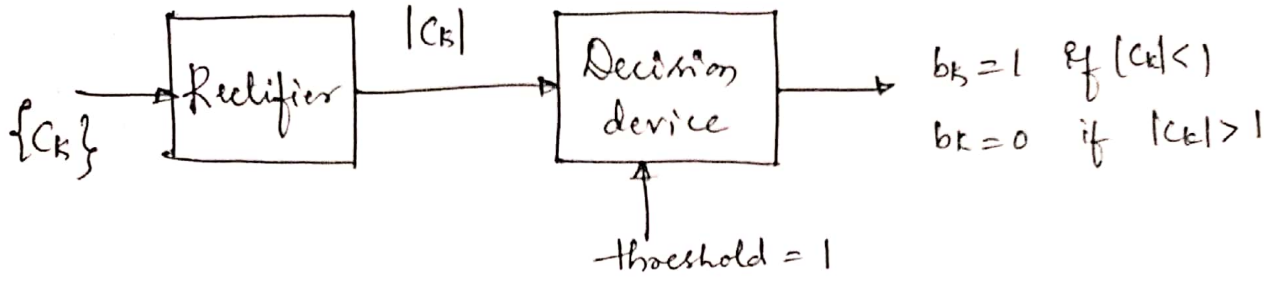


Precoder

The value of C_k will be,

$$C_k = \begin{cases} \pm 2 & \text{if } b_k = 0 \\ 0 & \text{if } b_k = 1 \end{cases}$$

Decoder :- Fig below shows the block diagram of duobinary decoder.



The magnitude of the o/p sequence c_k is taken. It is compared with '1'. Then decisions are taken as follows,

If $|c_k| > 1$ then $b_k = 0$

$|c_k| < 1$ then $b_k = 1$.

The o/p 'b_k' depends upon the present value of c_k. Since previous value of the o/p is not required. There is no propagation of error in the system.

Example :-

Sequences k	-1	0	1	2	3	4	5	6
1) Binary seq $\{b_k\}$		0	0	1	0	1	1	0
2) Precoded seq d_k	d_{-1} 1	1	1	0	0	1	0	0
3) polar representation of d_k $\{A_k\}$	+1	+1	+1	-1	-1	+1	-1	-1
4) Duo binary o/p c_k	-	+2	+2	0	-2	0	0	-2
5) Mag of $\lvert c_k \rvert$		+2	+2	0	+2	0	0	+2
6) o/p binary seq b_k		0	0	1	0	1	1	0

* Modified Duo binary (MDB) Encoding :-
(without Precoder)

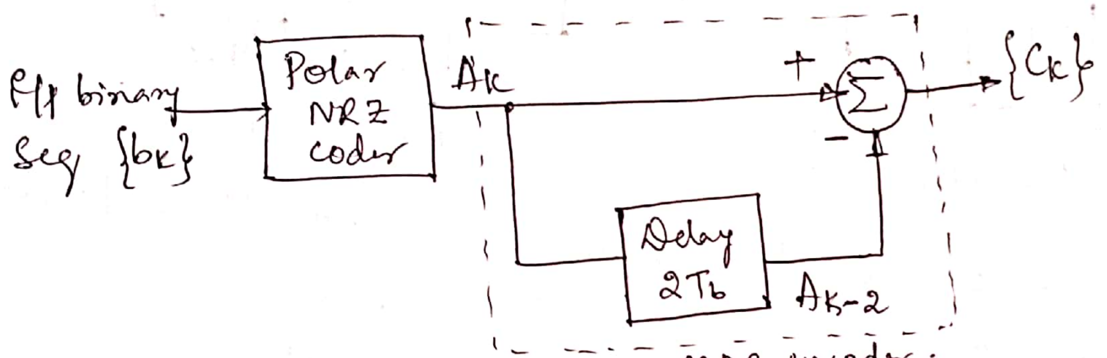
The correlation is over two binary digits in modified duo binary encoding. The output of the MDB is

$$\boxed{C_K = A_K - A_{K-2}}$$

where
$A_K = +1V$, if $b_K = 1$

$A_K = -1V$ if $b_K = 0$

Here the uncorrelated sequence $\{A_K\}$ of two level is converted into correlated sequence of three level pulses.



MDB encoder.

Reconstruction : Let $\hat{A}_K$ represents the estimate of A_K, then we can write,

$$\hat{A}_K = C_K + \hat{A}_{K-2}$$

This shows that if C_K is received with error, then $\hat{A}_K$ will have error. This error will propogate in the output sequence.

* MDB encoder with Precoder :-

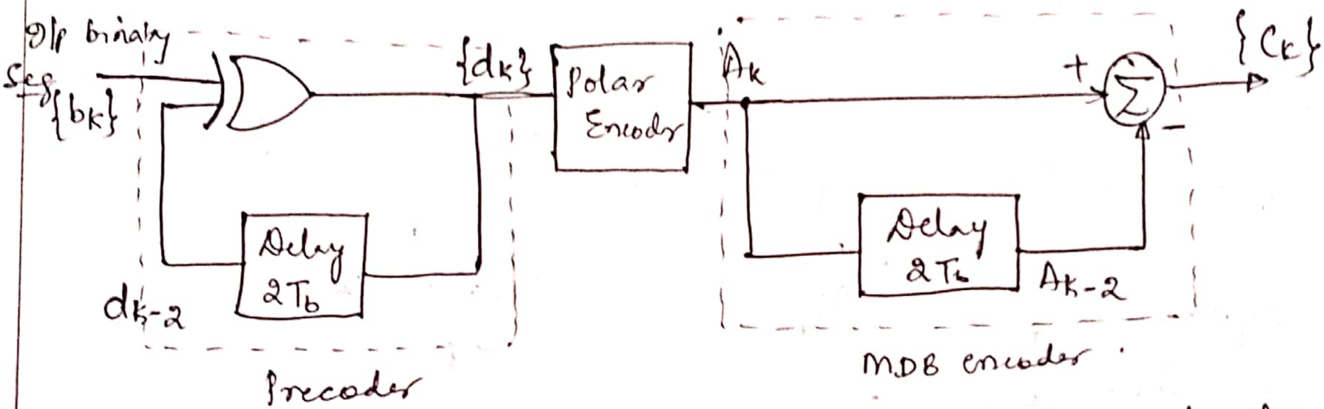
Encoder :- To avoid error propogation, a precoder (differential encoder) is used at the input. Fig shows the block diagram of MDB encoder with precoder.

The output of the precoder is

$$\boxed{d_k = b_k \oplus d_{k-2}}$$

This is normal Ex-OR operation. The precoded seq $\{d_k\}$ is applied to NRZ level encoder. The seq $\{A_k\}$ is then applied to MDB encoder. The o/p C_k is

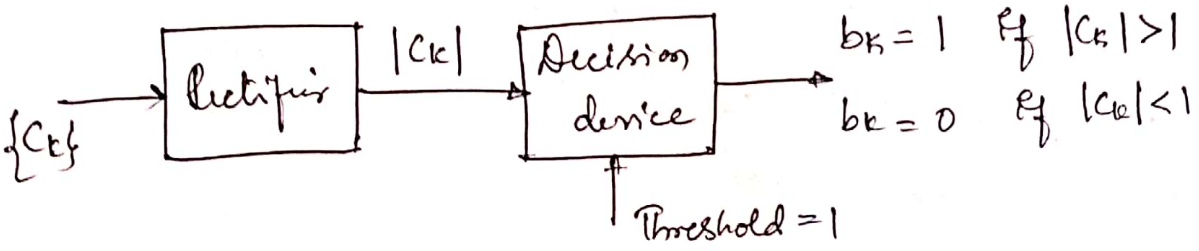
$$\boxed{C_k = A_k - A_{k-2}}$$



The sequence C_k is uncorrelated with three levels +2, 0, & −2.

$$C_k = \begin{cases} \pm 2, & \text{if } b_k = 1 \\ 0 & \text{if } b_k = 0 \end{cases}$$

<u>Decoder</u> :- Fig below shows the block diagram of MDB decoder.



The magnitude of the o/p sequence C_k is taken. It is compared with 1. Then decisions are taken as follows,

if $|C_k| > 1$ then $b_k = 1$

$|C_k| < 1$ then $b_k = 0$

Thus the precoding of the data at the transmitter makes it possible to detect the received data on a symbol-by-symbol basis without having to look back at previously detected symbols. Thus, error propagation is avoided.

Note:- Correlative coding (with precoder) is also called as symbol-by-symbol detection of data with controlled ISI.

Eye Diagram :- [Eye pattern]

The Eye pattern is used to study the effect of ISI in baseband digital transmission.

Consider the transmission of binary data 101101 by NRZ polar signalling [fig (a)]. If the channel is ideal with infinite bandwidth, pulses will be received without distortion. But because of channel distortion, the o/p will be distorted. If we cut the signal y(t) shown in fig (b) in each interval T_b & place it over one another, then we obtain the pattern as shown in fig (c). This pattern is called an eye pattern because of its resemblance to the human eye. The interior region of the eye pattern is called the eye opening.

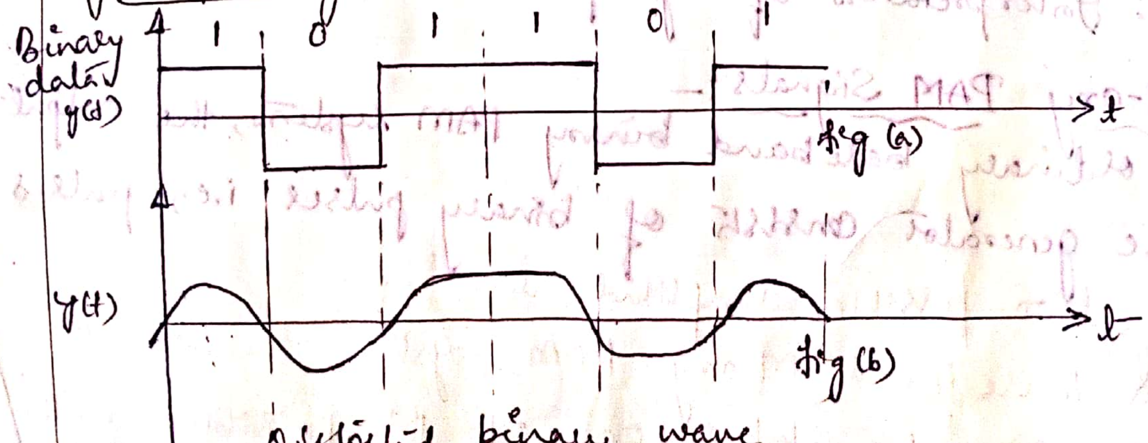


fig (a)

fig (b)

Distorted binary wave

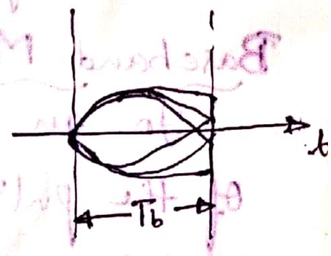


fig (c)

Eye pattern.

The eye pattern will provide the foll^g information,

1) The width of the eye opening defines the interval over which the received wave can be sampled without error from intersymbol interference. It is preferable to sample the instant at which eye is open widest.

2) The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied.

3) The height of the eye opening, at a specified sampling time, defines the margin over noise.

As the effect of ISI increases, the eye opening reduces. If the eye ~~opening~~ is closely completely, then it is not possible to avoid errors in the output.

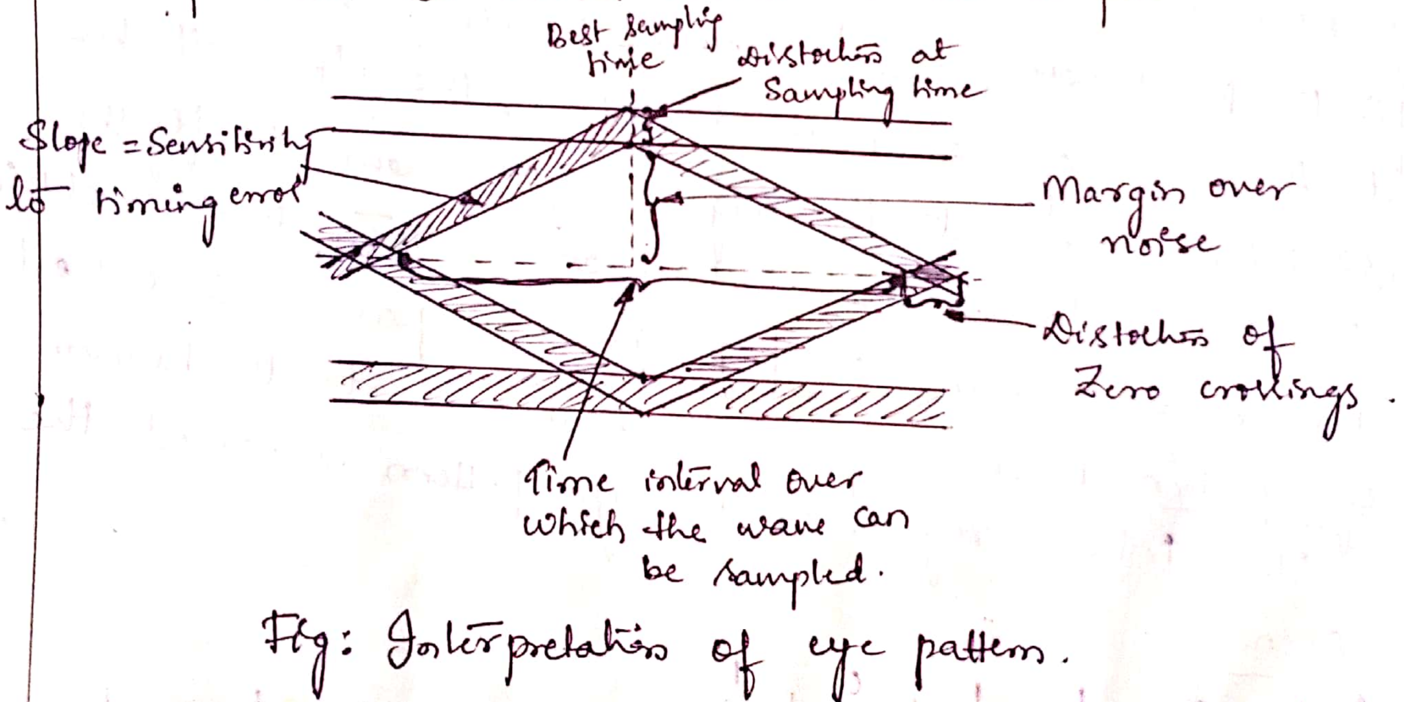


Fig: Interpretation of eye pattern.